A fluxgate magnetometer and an EMIS algorithm to study Europa's subsurface.

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Abstract

Magnetic field measurements on the surface of Europa can provide evidence of an internal salt-water ocean, and also help to characterize some of the subsurface properties. Observations from prior spacecraft such as Galileo and Ulysses have confirmed the existence of an induced magnetic field caused by the interaction of the Jovian magnetosphere with Europa's conductive medium. However, the thickness of the ice layer that separates the internal ocean from the surface has not been determined accurately based only on the measurements from orbit. Robotic systems capable of obtaining in situ measurements on the surface of Europa are being proposed for future space missions to this Jovian moon.

This report presents the development and implementation of a simulation model for a fluxgate magnetometer, as well as the Jovian magnetic field (VIP4). A fluxgate magnetometer can serve as a valuable scientific instrument for a future robotic mission to Europa, and it is well best suited for a penetrator mission carrying a small instrument package. In addition, an algorithm for electromagnetic induction sounding (EMIS) was developed as a software tool for estimating the thickness of the ice crust on Europa. Integration and alidation of the simulation models presented in this report is to be conducted as a future work.

1. Introduction

Jupiter's magnetic field model was coded and implemented to generate magnetic field data outputs, which can be used as the measurements taken by a fluxgate magnetometer, assuming the noise in the readings is zero. A simple fluxgate magnetometer model was developed and tested using Simulink, and outputs a 12-bit digital signal. In addition, an electromagnetic induction sounding (EMIS) algorithm was developed to estimate the thickness of the ice layer based on the apparent conductivity measured by the fluxgate magnetometer at the surface. Both the VIP4 simulation model, and the EMIS algorithm were developed using MATLAB™. The results and the issues encountered are presented and discussed in the following sections.

2. Nomenclature

H_J	Jupiter's Magnetic H-Field
R_J	Jupiter's Equatorial Radius
P_n^m	Legendre Function of degree *n* and order *m*
g_n^m	Schmidt cosine coefficient
h_n^m	Schmidt sine coefficient
ϑ	Latitude coordinates
ϕ	Longitude coordinates
μ_o	Permeability of free space
μ_a	Apparent Permeability
μ_r	Relative Permeability
ε_o	Permittivity of free space
B	Magnetic B-Field
E	Electric Field
V	Scalar Magnetic Potential
D	Electric Displacement Field

3. Technical Background

3.1. The VIP4 Jovian Magnetic Field Model

In situ measurements of the Jovian magnetic field were obtained between 1974 and 1995 during the flybys of spacecraft such as Pioneer 10 and 11, Voyager 1 and 2, Ulysses, and Galileo [1]. Different Jovian magnetic field models have been proposed based on the observations from orbit. However, the most recent and acceptable version of the magnetic field model is the VIP4 spherical harmonic model.

First and foremost, a local planetary magnetic field vector is obtained by taking the negative gradient of a scalar magnetic potential *V*, as shown in equation 1.

$$H_J = -\nabla V \tag{1}$$

The spherical harmonic expansion of *V* is given by equation 2:

$$V = R_J \sum_{n=1}^{n_{\max}} \left(\frac{R_J}{r}\right)^{n+1} \sum_{m=0}^{n} \left\{ P_n^m(\cos\theta)\left[g_n^m \cos(m\phi) + h_n^m \sin(m\phi)\right]\right\} \tag{2}$$

The equatorial radius of Jupiter R_J is about 71,380 km. Furthermore, *r* corresponds to the radial distance measured with respect to the centre of the planet. The list of the Schmidt coefficients of degree *n* and order *m* for the VIP4 model are given in Table 1.

Table 1: Internal Field Parameters for the Jovian VIP4 Magnetic Field

Parameter	Schmidt Coefficient	1974-1996 VIP4 [gauss]
1	g_1^0	4.205
2	g_1^1	-.659
3	h_1^1	0.250
4	g_2^0	-.051
5	g_2^1	-.619
6	g_2^2	0.497
7	h_2^1	-.361
8	h_2^2	0.053
9	g_3^0	-.016
10	g_3^1	-.520
11	g_3^2	0.244
12	g_3^3	-.176
13	h_3^1	-.088
14	h_3^2	0.408
15	h_3^3	-.316
16	g_4^0	-.168
17	g_4^1	0.222
18	g_4^2	-.061
19	g_4^3	-.202
20	g_4^4	0.066
21	h_4^1	0.076
22	h_4^2	0.404
23	h_4^3	-.166
24	h_4^4	0.039

3.2. Principles of a Fluxgate Magnetometer

A fluxgate magnetometer comprises primarily of a nonlinear ferromagnetic material surrounded by a primary and secondary winding set of coils, which are used to measure an induced voltage [2]. A simple representation of a fluxgate magnetometer sensing core is shown in Figure 1.

An alternating current drives the material periodically until it reaches its saturation point, inducing a voltage in the sensing coils. Odd harmonics are introduced in the induced voltage if no external magnetic field is present. However, in the presence of an external magnetic field, odd harmonics are added to the previous voltage signal. The intensity of this external field can be deduced by analyzing the voltage signal harmonics. Data processing techniques can be used for this purpose.

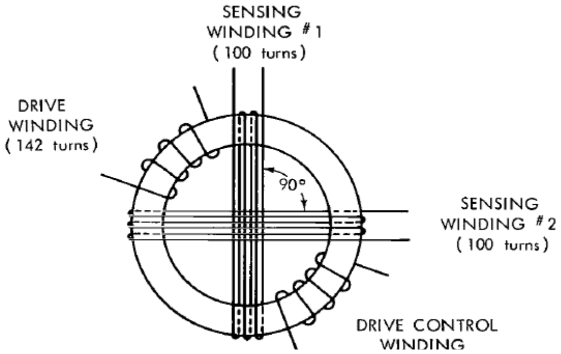


Figure 1: Conceptual design of a fluxgate magnetometer sensing core [2]

The most common type of fluxgate magnetometer is the parallel detector shown in Figure 1. As previously stated, the excitation coils supply an alternating current that produces a magnetic field, which changes the permeability of the core, and therefore saturates it. The relative permeability of a material is given by equation 3.

$$\mu_r = \frac{\mu}{\mu_o} \tag{3}$$

The apparent permeability of a material is given by equation 4:

$$\mu_a = \frac{\mu_r}{1 + D(\mu_r - 1)} \tag{4}$$

where, $\mu_o = 4\pi \times 10^{-7}$ is the permeability of free space and D is the electric displacement field, which relates the permittivity of a medium with electric field. For a linear, isotropic material, this relation is given by equation 5.

$$D = \varepsilon E \tag{5}$$

The induced voltage in the sensing coils is obtained by an approximation of Faraday's Law , as shown in equation 6:

$$V_{sns} = -N_{sns} A_{sns} \frac{dB}{dt} \tag{6}$$

The subscript *sns* is added to the parameters associated with the *sensing coils.* Where N_{sns} is the number of turns and A_{sns} is the total area . The magnetic H-field inside the ferromagnetic core is given by equation 7.

$$H = \frac{H_{ext}}{1 + D(\mu_r - 1)} \tag{7}$$

Similarly, the magnetic B-field sensed at the core can be expressed by equations 8 and 9:

$$B = \frac{\mu_o \mu_r}{1 + D(\mu_r - 1)} H_{ext} \tag{8}$$

$$B = \frac{\mu_r}{1 + D(\mu_r - 1)} B_{ext} \tag{9}$$

3.3. Electromagnetic Induction in Europa

The principle of electromagnetic induction states that if a conductive material is placed in a time-varying primary magnetic field, eddy currents flow on the surface of the conductor in response to the accompanying varying electric field [4]. As a result of the eddy currents, a secondary or induced magnetic field is generated, which reduces the primary field inside the conductor. This interaction is particularly evident in Europa, which is believed to have a conductive salt-water ocean beneath its ice layer.

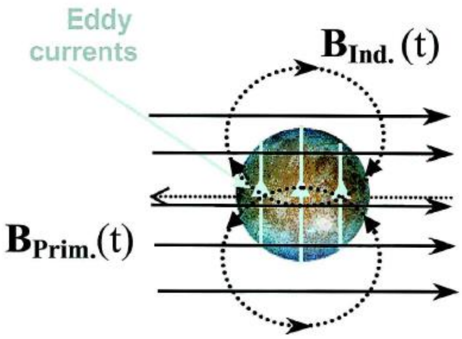
The addition of the Jovian magnetic field and the induced magnetic field on the surface of Europa results in the total magnetic field, which is given by equation 10.

$$B_{total} = B_J + B_{ind} \tag{10}$$

Furthermore, Europa's induced magnetic field can be determined by using equation 11 (Parkinson, 1983):

$$B_{ind} = \frac{\mu_o}{4\pi}\left[3(r \cdot M)r - r^2 M\right] / r^5 \tag{11}$$

where, *M* is the induction moment oscillating on the opposite direction of the Jovian magnetic field [5]. A representation of the electromagnetic induction in Europa is illustrated in Figure 2.



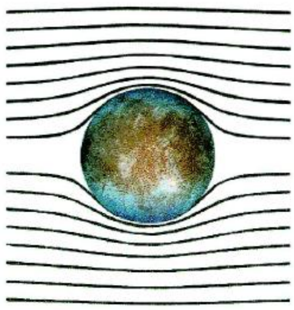


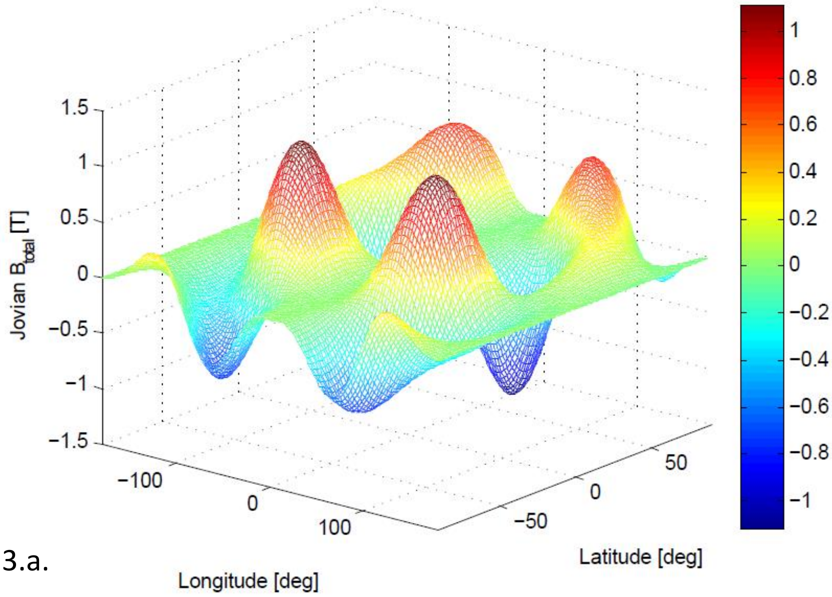
Figure 2: The figure on the top shows the time-varying Jovian magnetic field (solid lines) and the induced field (dotted lines) produced by the eddy currents. The bottom shows how the sum of the Jovian magnetic field and the induced fields create a force field to avoid the conducting obstacle (salt-water ocean) [4]

4. Simulation Results

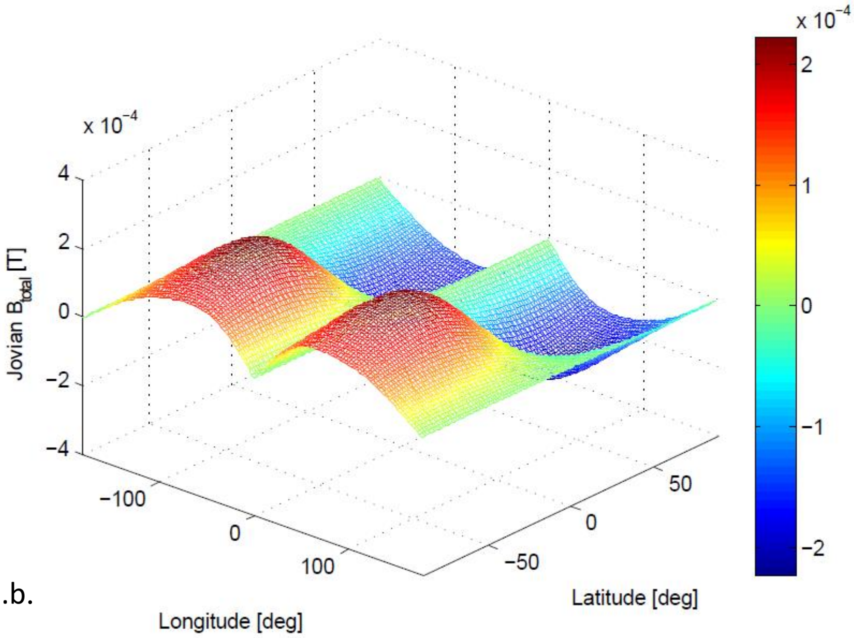
4.1. Implementation of the VIP4 Magnetic Field in MATLAB™

Based on the mathematical model presented in section 3.1, an algorithm was developed and implemented to simulate the VIP4 magnetic field data at any given location in space with respect to Jupiter's reference frame. The multi-pole series expansion and magnetic field simulation are performed by a MATLAB™ based algorithm developed for this project.

The results of the Jovian magnetic field intensity variation at the surface of the planet is illustrated in Figure 3-a. Furthermore, at a distance of 9.4(R_J), which is equivalent to the average orbital radius of Europa, the variation in magnetic field intensity is less intense. The results for this particular scenario is shown in Figure 3-b.



3.a.



3.b.

Figure 3: a. Mapping of the VIP4 Jovian magnetic B-field model on the surface of Jupiter ($R=R_J$). **b.** Mapping of the VIP4 magnetic field at a distance $R=9.4R_J$.

Based on the mathematical background presented in section 3.2, a fluxgate magnetometer model was developed in Simulink. In the absence of some parameters such as the induction moment, the following assumptions were made:

- The induced magnetic field on Europa's surface is assumed to be uniform with a constant magnitude of 100nT
- The relative permeability of the selected ferromagnetic material is assumed to be time-invariant

The selected design specifications for the fluxgate magnetometer are summarized in Table 2.

Table 2: Design Specifications for the Fluxgate Magnetometer Model

Parameter	**Value**	**Units**
Permeability of Ferrite	$1x10^{-4}$	F/m
Relative Permeability	79.6	-
Permittivity of MnZn Ferrite	$1x10^{5}$	N/Am^{2}
# Coil Turns	100	-
Area	$2x10^{-4}$	m^{2}

The Simulink block diagram of the proposed fluxgate magnetometer design is shown in Figure 4.

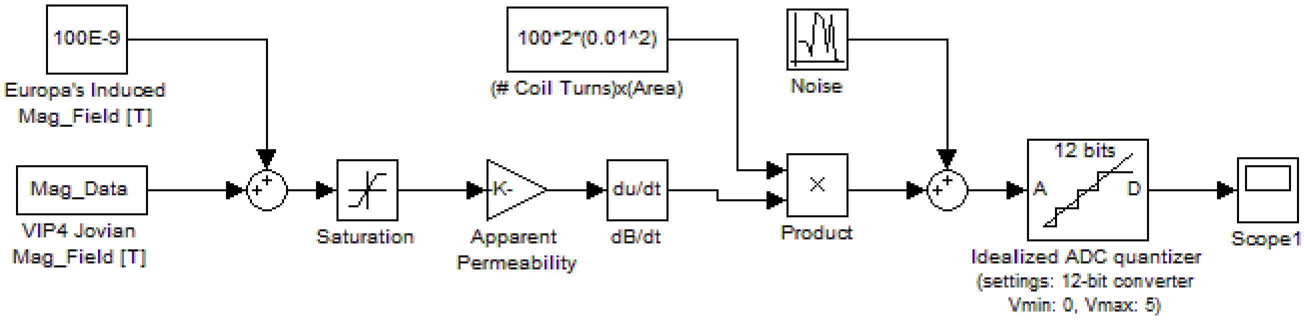


Figure 4: Simulink Block Diagram of the Fluxgate Magnetometer Model

4.3. EMIS Algorithm for Ice Thickness Estimation on Europa

The EMIS algorithm was developed using MATLAB™ as a simple function of the VIP4 Jovian magnetic field and the induced magnetic field on Europa. First and foremost, the algorithm employs a simple formula for the apparent conductivity of the medium (see equation 12) [6] [7].

$$\sigma_a = \frac{4}{\omega \mu_o r^2} \frac{H_{ind}}{H_J} \tag{12}$$

where, r is the radial distance from Europa to the centre of Jupiter. The distance from the bottom of the ice layer to the receiver of the magnetometer would be given by equation 13:

$$t = s_1 - \ln(\sigma_a - s_2)/s_3 \tag{13}$$

where, the s_n terms are the *Solórzano Coefficients*, which could be determined empirically. Equation 13 provides an approximation of the thickness of the ice layer that separates the liquid salt-water ocean from the surface. For the purpose of validation, a Montecarlo simulation of uniformly distributed magnetic field data points on the surface of Europa was performed. The discrete data points were fed into the EMIS function to estimate the ice layer thickness. The results of this simulation are plotted in Figure 5.

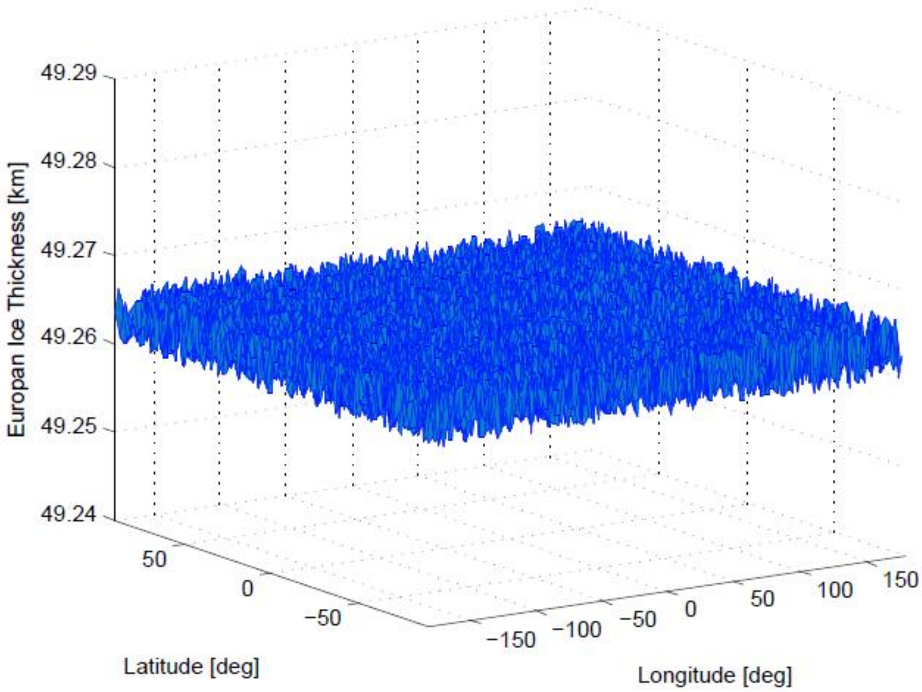


Figure 5: EMIS simulation of the ice-layer thickness variation on the surface of Europa

5. Future Work

Some suggestions to validate and improve the functionality of the simulation models and algorithms include but are not limited to:

- Compare and evaluate the performance of the existing MATLAB^{TM} based VIP4 model to state-of-the-art tools such as the Jupiter Environment Tool (JET) developed by NASA's JPL as a plugin for AGI Satellite Toolkit (STK).
- Integrate the models and algorithms into a single simulation package, and create a Graphical User Interface (GUI) to facilitate the operation
- Possible prototyping of the fluxgate magnetometer, testing, and analog experimentation in regions such as the Arctic or Antarctica.

6. Conclusion

A simulation model of a basic fluxgate magnetometer was presented in this report. Some discrepancies were observed between the simulated VIP4 Jovian magnetic field data and the actual average values observed by NASA spacecraft. For this reason, the simulation models and algorithms still require substantial improvement and revision. When these tools have are validated completely, they can be integrated into a single software package . Furthermore, the applicability of these tools can be expanded to other planets and moons within our solar system.

7. Acknowledgements

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